

Particle-like solutions to higher order curvature Einstein–Yang–Mills systems in d dimensions

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Abstract

We consider the superposition of the first two members of the gravitational hierarchy (Einstein plus first Gauss-Bonnet(GB)) interacting with the superposition of the first two members of the $SO_{(\pm)}(d)$ Yang–Mills hierarchy, in d dimensions. Such systems can occur in the low energy effective action of string theory. Particle-like solutions in dimensions $d = 6, 8$ are constructed respectively. Our results reveal qualitatively new properties featuring double-valued solutions with critical behaviour. In this preliminary study, we have restricted ourselves to one-node solutions.

1 Introduction

Gravitational theories in higher dimensions are of current interest in the contexts of the AdS/CFT correspondence and of theories with large and infinite extra dimensions with non-factorisable metrics. Since non-Abelian gauge fields feature in the low energy effective action of string theory, it is interesting to study the properties of the corresponding Einstein–Yang–Mills (EYM) systems. It is our purpose in the present work, to study higher dimensional EYM systems extended by higher order terms in both gravitational and YM curvatures.

In particular, we construct static particle-like solutions in the higher curvature version of Einstein–Yang–Mills systems in d -dimensional spacetimes in which the $SO(d)$ gauge field takes its values in the chiral representation $SO(d)_\pm$. This is analogous to the 4 dimensional model with $SO_\pm(4) = SU_\pm(2)$ Yang–Mills (YM) field interacting with Einstein gravity, in the work of Bartnik and McKinnon [1].

In the original system studied in [1], the gravitational action provides the required scaling behaviour needed to balance the scaling of the static Yang–Mills (YM) system in 3 Euclidean dimensions. This scaling argument is not rigorous because the gravitational action is not positive definite by construction, but nonetheless it enables the construction of particle like solutions. As a result these solutions are unstable and can be viewed as sphalerons[2] rather than solitons. The $(d - 1)$ static solutions we construct here are expected to have such an instability, though in this preliminary work we will not carry out the corresponding stability analysis. We will restrict to the simplest model incorporating only up to the second order curvature terms in both the YM and gravitational sectors.

As it happens, to go to $8 \geq d > 4$ it is necessary to include the *second* member of the YM hierarchy[3] to provide the requisite scaling, even in the absence of the gravitational GB term. (Similarly for $12 \geq d > 8$ it is necessary to include the *third* member of the YM hierarchy.) We will restrict our considerations to the first *two* members of the YM hierarchy, and hence to $d \leq 8$. Thus we will consider systems (a) featuring only Einstein gravity, (b) only GB gravity, and (c) mixed Einstein and GB gravity, but always with the first *two* members of the YM hierarchy.

In section 2, we introduce the models to be studied and in section 3 we derive the classical equations subject to our spherically symmetric Ansatz. This consists of the d -dimensional analogue of the 4-dimensional metric Ansatz employed in [1, 2], and the spherically symmetric Ansatz for the YM field in the $SO_\pm(d)$ (chiral) representation in d -dimensions, analogous with the Ansatz for the $SU(2)$ YM field in the $SO_\pm(4)$ (chiral) representation in 4-dimensions used in [1, 2]. Section 4 and its subsections are devoted to our numerical study of these systems and we give a summary and discuss our results in section 5.

2 The model(s)

The generic system of interest in this work is the superposed Yang–Mills hierarchy [3] interacting with the superposed gravitational hierarchy consisting of the Einstein and all possible Gauss–Bonnet (GB) terms in a given spacetime dimension d .

$$\mathcal{L} = \mathcal{L}_{grav} + \mathcal{L}_{YM} . \quad (1)$$

In this preliminary work, we will carry out this construction for dimensions $d = 6, 8$ only.

The definition we use for the superposed gravitational hierarchy is

$$\mathcal{L}_{grav} = \sum_{p=1}^{P_1} \frac{1}{2^p} \kappa_p e R_{(p)} , \quad (2)$$

in which the $p = 1$ term describes the usual Einstein gravity, and the higher p terms the successive GB contributions. In (2) $e = \det e_\mu^a = \sqrt{\det g_{\mu\nu}}$, e_μ^a are the vielbeins, and $R_{(p)}$ is the p -Ricci scalar. In terms of the totally antisymmetrised p -fold product of the Riemann tensor $R_{\mu\nu}^{ab}$ yielding the $2p$ -form p -Riemann tensor (with the notation $[abc\dots]$ implying total antisymmetrisation of the indices a, b, c, \dots)

$$R_{\mu_1\mu_2\dots\mu_{2p}}^{a_1a_2\dots a_{2p}} = R_{[\mu_1\mu_2}^{a_1a_2} R_{\mu_3\mu_4}^{a_3a_4} \dots R_{\mu_{2p-1}\mu_{2p}}^{a_{2p-1}a_{2p}}] , \quad (3)$$

we define the p -Ricci scalar and the p -Ricci tensor as

$$R_{(p)} = R_{\mu_1\mu_2\dots\mu_{2p}}^{a_1a_2\dots a_{2p}} e_{a_1}^{\mu_1} e_{a_2}^{\mu_2} \dots e_{a_{2p}}^{\mu_{2p}} \quad (4)$$

$$R_{(p)\mu}^a = R_{\mu\mu_1\mu_2\dots\mu_{2p-1}}^{aa_1a_2\dots a_{2p-1}} e_{a_1}^{\mu_1} e_{a_2}^{\mu_2} \dots e_{a_{2p-1}}^{\mu_{2p-1}} , \quad (5)$$

Leading to the definitions of the p -Einstein tensor

$$G_{(p)\mu}^a = R_{(p)\mu}^a - \frac{1}{2p} e_\mu^a R_{(p)} . \quad (6)$$

Note that the p -Ricci scalar (4) coincides with the Euler-Hirzbruch density¹ in $d = 2p$ (even) dimensions, and hence is trivial. In dimensions $d \leq 2p$ on the other hand, it vanishes by (anti)symmetry irrespective if d is even or odd. Thus the upper limit in the summation in (2) is $P_1 = \frac{d-2}{2}$ in even, and $P_1 = \frac{d-1}{2}$ in odd dimensions d .

The definition we use for superposed YM hierarchy is

$$\mathcal{L}_{YM} = \sum_{p=1}^{P_2} \frac{1}{2(2p)!} \tau_p e \text{Tr} F(2p)^2 , \quad (7)$$

where $F(2p)$ is the $2p$ -form p -fold totally antisymmetrised product of the $SO(d)$ YM curvature 2-form $F(2)$

$$F(2p) \equiv F_{\mu_1\mu_2\dots\mu_{2p}} = F_{[\mu_1\mu_2} F_{\mu_3\mu_4} \dots F_{\mu_{2p-1}\mu_{2p}}] . \quad (8)$$

Even though the $2p$ -form (8) is dual to a total divergence, namely the divergence of the corresponding Chern-Simons form, the density (7) is never a total divergence since it is the square of one. But the $2p$ -form (8) vanishes by (anti)symmetry for $d < 2p$ so that the upper limit in the summation in (7) is $P_2 = \frac{d}{2}$ for even d and $P_2 = \frac{d-1}{2}$ for odd d .

We define the p -stress tensor pertaining to each term in (7) as

$$T_{\mu\nu}^{(p)} = \text{Tr} F(2p)_{\mu\tau_1\tau_2\dots\tau_{2p-1}} F(2p)_{\nu}{}^{\tau_1\tau_2\dots\tau_{2p-1}} - \frac{1}{4p} g_{\mu\nu} \text{Tr} F(2p)_{\tau_1\tau_2\dots\tau_{2p}} F(2p)^{\tau_1\tau_2\dots\tau_{2p}} . \quad (9)$$

In the present work, we restrict our attention to $d = 6$ and 8 , and in both cases restrict to only the two terms $p = 1$ and 2 , both in the gravitational and the YM hierarchies, even though in these dimensions higher values of P both in (2) and (7) can be accommodated. This restriction is motivated by our desire to study the simplest nontrivial cases. As for our choice of $d = 6$ and 8 , these are the only even dimensions in which the system consisting of (2) and (7) is nontrivial, and our restriction in this preliminary study is motivated by our desire to maximise the analogy with the study of Bartnik and McKinnon [1] which is in (even) $d = 4$. Like in the model of [1], we will choose the YM gauge group to be $SO(d)$, with the YM fields taking their values in the chiral representation $SO_\pm(d)$ (which for $d = 4$ reduces to $SU_\pm(2)$).

¹This follows immediately from the alternative definition for the p -Ricci scalar in d dimensions:

$$\hat{R}_{(p)} = \varepsilon^{\nu_1\nu_2\dots\nu_{2p}\mu_1\mu_2\dots\mu_{d-2p}} e_{\mu_1}^{a_1} e_{\mu_2}^{a_2} \dots e_{\mu_{d-2p}}^{a_{d-2p}} \varepsilon^{b_1b_2\dots b_{2p}a_1a_2\dots a_{d-2p}} R_{\nu_1\nu_2}^{b_1b_2} R_{\nu_3\nu_4}^{b_3b_4} \dots R_{\nu_{2p}\nu_{2p}}^{b_{2p}b_{2p}} .$$

3 The classical equations

In d dimensional spacetime, we restrict to static fields that are spherically symmetric in the $d - 1$ spacelike dimensions with the metric Ansatz

$$ds^2 = -A(r)dt^2 + B(r)^{-1}dr^2 + r^2 d\Omega_{d-2}^2 \quad (10)$$

where r is the $(d - 2)$ dimensional spacelike radial coordinate and $d\Omega_{d-2}$ is the $d - 2$ dimensional angular volume element.

We take the static spherically symmetric $SO(d)$ YM field in even d to be in the $SO_{(\pm)}(d)$ (chiral) representation

$$A_0 = 0, \quad A_i = \left(\frac{w - 1}{r} \right) \Sigma_{ij}^{(\pm)} \hat{x}_j, \quad \Sigma_{ij}^{(\pm)} = -\frac{1}{4} \left(\frac{1 \pm \Gamma_{d+1}}{2} \right) [\Gamma_i, \Gamma_j]. \quad (11)$$

The index $i = 1, 2, \dots, d - 1$ labels the space dimensions of the static fields, Γ_μ ($\mu = 1, 2, \dots, d$) being the gamma matrices in d dimensions. $\Sigma_{ij}^{(\pm)}$ are the generators of the $SO(d - 1)$ subgroup of $SO(d)$, the latter being in the chiral representation. We note that (11) is not the most general spherically symmetric Ansatz for a $SO(d)$ YM connection and is effectively that of a $SO(d - 1)$ field. In this sense we could regard our gauge fields to be $SO(d - 1)$ rather than $SO(d)$ fields. In the low dimensional case $d = 4$, the generators $\Sigma_{\mu\nu}^{(\pm)}$ reduce to the those of $SU_+(2)$ and $SU_-(2)$, in which case (11) is indeed general enough. Another difference of $d > 4$ versus $d = 4$ is that in the latter case there is a duality between the electric and magnetic fields while in our case this duality is absent. Thus we do not seek to construct dyonic particle like solutions and have set $A_0 = 0$ in (11).

For the system (1) under consideration, namely that with $P_1 = P_2 = 2$, the variational equations for the YM and gravitational fields are, respectively

$$\tau_1 D_\mu (e F^{\mu\nu}) + \frac{1}{2} \tau_2 \{F_{\rho\sigma}, D_\mu (e F^{\mu\nu\rho\sigma})\} = 0 \quad (12)$$

$$\kappa_1 G_{(1)\mu}^a + \kappa_2 G_{(2)\mu}^a = \tau_1 T_{(1)\mu}^a + \frac{1}{3!} \tau_2 T_{(2)\mu}^a. \quad (13)$$

It is easy to systematically extend (12) and (13) to the highest values of P_1 and P_2 respectively, in any dimension d .

By virtue of the spherically symmetric YM Ansatz (11) and the metric Ansatz (10), it follows that the Gauss Law equation, namely the $\nu = 0$ (or $\nu = t$) component of (12) is identically satisfied. Amongst the $\nu = i$ components, the $\nu = r$ also trivialise and the $d - 2$ equations pertaining to the $\nu = \theta_\alpha$ components all coincide to give only one equation for (11). We list explicitly the two terms in (11)

$$D_\mu (e F^{\mu\theta_\alpha}) = (r^{d-4} \sqrt{AB} w')' - (d - 3) r^{d-6} \sqrt{\frac{A}{B}} (w^2 - 1) w \quad (14)$$

$$\begin{aligned} \{F_{\rho\sigma}, D_\mu (e F^{\mu\theta_\alpha\rho\sigma})\} &= 3(d - 3)(d - 4)(w^2 - 1) \left((r^{d-8} \sqrt{AB} (w^2 - 1) w')' \right. \\ &\quad \left. - (d - 5) r^{d-10} \sqrt{\frac{A}{B}} (w^2 - 1)^2 w \right) \end{aligned} \quad (15)$$

for all $\alpha = 1, 2, \dots, (d - 2)$.

It follows from the metric Ansatz (10) that the Einstein tensors defined in (6) have nonvanishing components $G_{(p)t}^t$, $G_{(p)r}^r$ and $G_{(p)\theta_\alpha}^{\theta_\alpha}$ ($\alpha = 1, 2, \dots, (d-2)$), which can readily be calculated for any p . Here we list explicitly the two cases of interest, $p = 1$ and $p = 2$, respectively,

$$G_{(1)t}^t = \frac{1}{2r}(d-2) \left[B' - \frac{1}{r}(d-3)(1-B) \right] \quad (16)$$

$$G_{(1)r}^r = \frac{1}{2r}(d-2) \left[\frac{BA'}{A} - \frac{1}{r}(d-3)(1-B) \right] \quad (17)$$

$$G_{(1)\theta_\alpha}^{\theta_\alpha} = \frac{1}{2} \left[\left(\frac{BA''}{A} + \frac{1}{2} \left(\frac{B}{A} \right)' A' \right) - \frac{1}{r^2}(d-3)(d-4)(1-B) \right. \\ \left. + \frac{1}{r}(d-3) \frac{(AB)'}{A} \right], \quad (18)$$

and

$$G_{(2)t}^t = \frac{1}{4r^3}(d-2)(d-3)(d-4)(1-B) \left[B' - \frac{1}{2r}(d-5)(1-B) \right] \quad (19)$$

$$G_{(2)r}^r = -\frac{1}{4r^3}(d-2)(d-3)(d-4)(1-B) \left[\frac{BA'}{A} - \frac{1}{2r}(d-5)(1-B) \right] \quad (20)$$

$$G_{(2)\theta_\alpha}^{\theta_\alpha} = \frac{1}{4r^2}(d-3)(d-4) \left[(1-B) \left(\frac{BA''}{A} + \frac{1}{2} \left(\frac{B}{A} \right)' A' \right) - \left(\frac{BA'}{A} \right) B' \right. \\ \left. + \frac{1}{r}(d-5)(1-B) \frac{(AB)'}{A} - \frac{1}{2r^2}(d-5)(d-6)(1-B)^2 \right]. \quad (21)$$

It follows from the Ansätze (10) and (11) that the stress tensors defined in (9) have nonvanishing components $T_{(p)t}^t$, $T_{(p)r}^r$ and $T_{(p)\theta_\alpha}^{\theta_\alpha}$ ($\alpha = 1, 2, \dots, (d-2)$), which can readily be calculated for any p . Here we list explicitly the two cases of interest, $p = 1$ and $p = 2$, respectively,

$$T_{(1)t}^t = -\frac{1}{2^4 r^2} n_d (d-2) \left[2Bw'^2 + (d-3) \left(\frac{w^2-1}{r} \right)^2 \right] \quad (22)$$

$$T_{(1)r}^r = -\frac{1}{2^4 r^2} n_d (d-2) \left[-2Bw'^2 + (d-3) \left(\frac{w^2-1}{r} \right)^2 \right] \quad (23)$$

$$T_{(1)\theta_\alpha}^{\theta_\alpha} = -\frac{1}{2^4 r^2} n_d \left[2(d-4)Bw'^2 + (d-3)(d-6) \left(\frac{w^2-1}{r} \right)^2 \right], \quad (24)$$

and

$$T_{(2)t}^t = -\frac{(3!)^2}{2^7 r^4} n_d \frac{(d-2)!}{(d-5)!} \left(\frac{w^2-1}{r} \right)^2 \left[4Bw'^2 + (d-5) \left(\frac{w^2-1}{r} \right)^2 \right] \quad (25)$$

$$T_{(2)r}^r = -\frac{(3!)^2}{2^7 r^4} n_d \frac{(d-2)!}{(d-5)!} \left(\frac{w^2-1}{r} \right)^2 \left[-4Bw'^2 + (d-5) \left(\frac{w^2-1}{r} \right)^2 \right] \quad (26)$$

$$T_{(2)\theta_\alpha}^{\theta_\alpha} = -\frac{(3!)^2}{2^7 r^4} n_d \frac{(d-3)!}{(d-5)!} \left(\frac{w^2-1}{r} \right)^2 \left[4(d-8)Bw'^2 + (d-5)(d-10) \left(\frac{w^2-1}{r} \right)^2 \right]. \quad (27)$$

(24) and (27) are valid for all $\alpha = 1, 2, \dots, (d-2)$. The constants $n_d = \text{Tr } \mathbb{I}$, where the dimensionality of the unit matrix is determined by the chiral representations appearing in (11).

Substituting (14)-(15) in (12) results in one equation, and substituting (16)-(27) in (13) results in three equations. Thus we have 4 ordinary differential equations for the 3 radial functions $A(r)$, $B(r)$ and $w(r)$. This necessitates the verification of the consistency of these 4 equations to ensure that the system is not overdetermined. We have carried out this consistency check in the case at hand with $P_1 = P_2 = 2$ in (2) in (7) respectively, for any $\kappa_1, \kappa_2, \tau_1, \tau_2$ and in any dimension d . We found that the components $(\theta_\alpha, \theta_\alpha)$ of (13),

$$\kappa_1 G_{(1)\theta_\alpha}^{\theta_\alpha} + \kappa_2 G_{(2)\theta_\alpha}^{\theta_\alpha} = \tau_1 T_{(1)\theta_\alpha}^{\theta_\alpha} + \frac{1}{3!} \tau_2 T_{(2)\theta_\alpha}^{\theta_\alpha} . \quad (28)$$

are identically satisfied by the two other Einstein equations

$$\kappa_1 G_{(1)t}^t + \kappa_2 G_{(2)t}^t = \tau_1 T_{(1)t}^t + \frac{1}{3!} \tau_2 T_{(2)t}^t \quad (29)$$

$$\kappa_1 G_{(1)r}^r + \kappa_2 G_{(2)r}^r = \tau_1 T_{(1)r}^r + \frac{1}{3!} \tau_2 T_{(2)r}^r \quad (30)$$

together with the YM equation (12). Thus the 3 effective constraints on the 3 radial functions are the 2 equations (29)-(30) plus the YM equation (12). This check of consistency extends by induction to the case of arbitrary P_1, P_2 .

Finally we express these 3 equations employing the same notation as [1] and [2], namely

$$A(r) = \sigma^2(r) N(r) \quad , \quad B(r) = N(r) . \quad (31)$$

For the functions $w(r)$, $N(r)$ and $\sigma(r)$, we have

$$\begin{aligned} & 2\tau_1 \left((r^{d-4} \sigma N w')' - (d-3)r^{d-6} \sigma (w^2 - 1)w \right) + \\ & + 3\tau_2 (d-3)(d-4)(w^2 - 1) \left((r^{d-8} \sigma N (w^2 - 1)w')' - (d-5)r^{d-10} \sigma (w^2 - 1)^2 w \right) = 0 \end{aligned} \quad (32)$$

$$\begin{aligned} m' &= \frac{1}{8} r^{d-4} \left(\tau_1 \left[N w'^2 + \frac{1}{2} (d-3) \left(\frac{w^2 - 1}{r} \right)^2 \right] \right. \\ & \quad \left. + \frac{3}{2} \frac{\tau_2}{r^2} (d-3)(d-4) \left(\frac{w^2 - 1}{r} \right)^2 \left[N w'^2 + \frac{1}{4} (d-5) \left(\frac{w^2 - 1}{r} \right)^2 \right] \right) \end{aligned} \quad (33)$$

$$\left[\kappa_1 + \frac{\kappa_2}{2r^2} (d-3)(d-4)(1-N) \right] \left(\frac{\sigma'}{\sigma} \right) = \frac{n_d}{8r} \left[\tau_1 + \frac{3}{2} \frac{\tau_2}{r^2} (d-3)(d-4) \left(\frac{w^2 - 1}{r} \right)^2 \right] w'^2 . \quad (34)$$

In (33), the function $m(r)$ is defined as

$$m(r) = n_d^{-1} [\kappa_1 r^{d-3} (1-N) + \frac{1}{4} \kappa_2 r^{d-5} (1-N)^2] , \quad (35)$$

which for $\kappa_2 = 0$ and with $d = 4$ reduces to the definition of the corresponding function $m(r)$ used in [1, 2]. For $\kappa_2 = \tau_2 = 0$ and with $d = 4$, equations (32)-(34) coincide with the ordinary differential equations of [1, 2].

4 Numerical results

In this section we present the results we obtained for the two cases corresponding to $p = 1$ and $p = 2$ gravity and for the corresponding space-time dimensions $d = 6$ and $d = 8$ where solutions of the extended Yang-Mills equations (consisting of the superposition of the $p = 1$ and $p = 2$ members of the YM hierarchy) exist even in the absence of gravity. (The cases of odd values of d will be addressed elsewhere). When the two mixing parameters τ_1, τ_2 do not vanish we can set them equal to particular values without losing generality. This can be done by an appropriate rescaling of the overall lagrangian density and of the radial variable. So we take advantage of this freedom and choose $\tau_1 = 1, \tau_2 = 1/3$ (this simplifies the numerical coefficients in the equations). The limit $\tau_1 \rightarrow 0$ is addressed separately.

4.1 Boundary conditions

We have solved the above equations with the appropriate boundary conditions for the radial functions $m(r)$, $\sigma(r)$ and $w(r)$ which guarantee the solution to be regular at the origin and to have finite energy. These boundary values are

$$m(0) = 0 \quad , \quad w(0) = 1 \quad , \quad (36)$$

at the origin, and

$$\lim_{r \rightarrow \infty} \sigma(r) = 1 \quad , \quad \lim_{r \rightarrow \infty} w(r) = -1 \quad . \quad (37)$$

at infinity. The condition on σ results in Minkowskian metric asymptotically.

Further analysis of the YM equation (32) yields

$$w(r \rightarrow \infty) = -1 + \frac{C}{r^{d-3}} \quad (38)$$

4.2 $p = 1$ gravity: Pure Einstein-Hilbert

We define here $\alpha^2 \equiv n_d/(8\kappa_1)$ so that $\alpha^2 = 0$ corresponds to the decoupling of gravity (and therefore to a flat space).

4.2.1 $d = 6$ case

The flat solution corresponding to the flat case [4] is characterized by an energy $M \approx 7.096$ (in the conventions used). This solution is smoothly deformed by gravity and exists up to a critical value $\alpha_m^2 \approx 0.1266$.

When α^2 increases, the mass of the gravitating solution decreases from $M \approx 7.096$ (flat case) down to $M \approx 6.56$; the function $N(x)$ develops a local minimum, say N_m which becomes deeper while gravity becomes stronger and the value $\sigma(0)$ decreases from one. These quantities are plotted in Fig.1 and the mass is reported on Fig. 2

Our numerical analysis strongly indicates that a second branch of regular solutions exists, also terminating at $\alpha^2 = \alpha_m^2$.

On this second branch the values $\sigma(0)$ and N_m increase monotonically with α^2 . The quantity $\sigma(0)$ reaches the value zero $\alpha^2 \rightarrow \alpha_c^2$, with $\alpha^2 \approx 0.07$, as illustrated by Fig. 1.

Accordingly, the metric becomes singular in this limit and no solutions seem to exist on this branch for lower values of α^2 .

The second branch of solutions therefore exist for $\alpha_c < \alpha^2 \leq \alpha_m^2$. The mass of the solution of the second branch is larger than the corresponding one on the first branch, as illustrated by Fig. 2.

The minimal value of $N(r)$ stays well above zero ($N_{min} \approx 0.17$) so that no horizon is approached. The situation contrasts with the case of the gravitating monopole in 3+1 dimensions [5, 6]. In both cases (the present and that of [5, 6]) there occur second branches of the solutions after critical values the respective parameters (α^2 in our case), these bifurcations are quite different in the two cases. The gravitating monopole bifurcates into a Reissner-Nordström black hole solution while the solutions studied here terminate into a solution presenting a singularity at the origin. Several components of the Riemann tensor are indeed polynomials in the field $\sigma(r)$.

For scaling reasons, the parameter τ_2 cannot vanish for finite energy solutions to exist. The numerical study of the equation with a varying τ_1 (with α^2 fixed) reveals that, in the limit $\tau_1 \rightarrow 0$, the solutions of the first branch converge to the vacuum solution (with mass equal to zero and with $w(x) = 1$). When the solutions on the second branch are examined in the same limit, it appears that they stop to exist at a critical value of τ_1 where the value $\sigma(0)$ becomes zero. In other words, it also terminates into a singular solution for a finite value of τ_1 and no regular solution seems to exist for $\tau_1 = 0$.

4.2.2 $d = 8$ case

The scenario is qualitatively similar to the one of the $d = 6$ case. Here the flat solution has $M \approx 25.6$, it gets deformed by gravity up to $\alpha^2 \approx 0.0022$ and the function $\sigma(r)$ is such that $\sigma(0)$ decreases with α^2 . At the same time $N(r)$ develops a local minimum which becomes deeper. Another branch of solution exists where both $\sigma(0)$ and N_{min} are lower than their counterparts on the first branch. These relevant quantities are plotted in Figs. 3 and 4. On Fig. 5 we present the profiles of the metric functions for several values of the parameters.

The numerical analysis suggests that this second branch persists up to $\alpha^2 = 0$ and that in the limit, $\lim_{\alpha^2 \rightarrow 0} \sigma(0) = 0$. As far as our numerical analysis indicates, the value N_m tends to a finite value in this limit so that there occurs no horizon.

We further investigated the two branches of solutions in the limit $\tau_1 \rightarrow 0$. It appears that the first branch solution tends to the vacuum while the second one tends to a non trivial solution with a finite mass.

4.3 $p = 2$ gravity: Pure Gauss-Bonnet

This case is not of any particular physical relevance but it is interesting to learn the peculiarities of the pure $p = 2$ gravity especially when the YM term is also the pure $p = 2$ member of the YM hierarchy in $d = 8$. Such a system supports a self-dual YM solution on double-self-dual gravitational background [7] when the metric has Euclidean signature, so it is interesting to contrast with the case of Minkowskian signature at hand.

4.3.1 $d = 6$ case

We define the constant $\beta^4 \equiv 4n_d/\kappa_2$ in this case. Again the flat solution gets deformed by the gravitating term and a gravitating solution is formed, characterized by $\sigma(0)$, N_m (see Fig. 6) and by the mass (see Fig. 7).

While both $\sigma(0)$, N_m decrease monotonically as functions of β^2 , the mass first decreases, reaches a minimum for $\beta^2 \approx 0.75$ and then starts to increase very rapidly with β^2 .

In the present case, it is very likely that the gravitating solution exists up to a maximal value of β^2 , say β_m^2 and that, when this limit is approached, N_m tends to zero, forming an horizon, while M tends to infinity. (Incidentally, r_0 the value of the radial variable where $w(r)$ crosses zero, exhibits

the same behaviour). The numerical integration of the equations for $\beta^2 > 0.87$ becomes difficult and we could not evaluate β^2 with a good accuracy.

Denoting r_m the value of r where the function $N(r)$ attains its minimum, we observe that the solutions possess a completely different behaviour in the regions $r > r_m$ and $r < r_m$.

The functions $m(r), \sigma(r), w(r)$ vary essentially on the interval $[0, r_m]$ and attain their asymptotic values $m(\infty), 1, -1$ for $r \simeq r_m$. They are essentially constant on the interval $r \in]r_m, \infty[$. This feature is illustrated in Fig. 8.

Our belief that the solution stops at some maximal value of β^2 is reinforced by the fact that our attempts to produce a second branch of solutions turned out to be unsuccessful.

A peculiarity of the equations is that fixing β^2 and τ_1 and varying τ_2 , is equivalent to a rescaling of the radial variable r . Fixing β^2 and decreasing $\tau_1 \rightarrow 0$ the numerical results strongly suggest that the solution tends to the vacuum.

4.3.2 $d = 8$ case

The pattern of solutions is similar to the ones in the $d = 6$ case. Here the mass attains its minimum at $\beta^2 \approx 0.55$ and again the gravitating solutions seem to exist up to $\beta^2 \approx 1.2$. The relevant quantities characterizing the solutions are plotted in Figs. 9 and 10. The behaviour of the solutions for β^2 approaching the critical value is illustrated on Fig. 11.

The absence of a solution for the model consisting purely of the $p = 2$ members of both gravitational and YM hierarchies for Minkowskian signature $d = 8$ at hand, contrasts with the selfdual (black hole) solutions [7] in the case of Euclidean signature. (Note that the $p = 1$ counterpart in Minkowskian $d = 4$ exists, namely the solutions of [1], along with the selfdual solutions.) It would be interesting to find out if the situation is different for Minkowskian signature black hole solutions in $d = 8$.

4.4 $p = 1 + p = 2$ gravity: Superposed Einstein and Gauss-Bonnet

We finally considered the equations for the case where the $p = 1$ and $p = 2$ gravities are superposed. We limited our analysis to the case where the $p = 2$ gravity coupling is slightly smaller (although not too small in order to see the effect of the superposition) than the $p = 1$ parameter. To fix the idea, we choose $\kappa_2 = (1/2)\kappa_1$. Our conclusion is that for $d = 6$ and $d = 8$ the qualitative properties of the solutions corresponding to $p=1$ are conserved, namely two branches of solutions are found. The maximal value α^2 increases slightly in the mixed case; for instance $\alpha_m^2 \approx 0.004$ for our choice of the parameter in the $d = 8$ case.

5 Conclusions

It appears that the difference of the solutions that we have constructed, from the those given in [1] for the usual EYM system, arises due to the presence of the additional dimensional constants κ_2 and τ_2 . These constants result in the presence of a nontrivial parameter in the equations, for instance the one we denoted as α^2 , which contains the gravitational coupling strength. By varying this parameter we were able to construct one or several branches of solutions for the cases considered. In this respect the system of equations studied is analogous to that of the gravitating monopole [6] where the VEV of the Higgs fields also gives rise to an additional dimensional parameter.

In this preliminary study, we have restricted ourselves to the construction of the simplest static spherically symmetric (in $d - 1$ spacelike dimensions) solutions. Natural extensions of the present

work include (i) the construction of solutions with *several nodes* in the YM function $w(r)$, and (ii) the construction of the black hole counterparts of our particle-like solutions. In the latter case these pertain to the fully interacting version of the *fixed gravitational background* solutions discussed in [7]. Another extension, (iii), concerns the deformation of our solutions due to the inclusion of a cosmological constant [8]. Finally, (iv) the stability analysis of our solutions can be carried out.

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Figure Captions

- Figure 1 The minimal value of the function N and the value $\sigma(0)$ are plotted as functions of α^2 for $p = 1$ gravity in $d = 6$.
- Figure 2 The mass of the solutions for $p = 1$ gravity in $d = 6$.
- Figure 3 *Idem* Fig. 1 in $d = 8$.
- Figure 4 *Idem* Fig. 2 in $d = 8$.
- Figure 5 The profiles for the metric functions $N(r)$, $\sigma(r)$ are presented for different values of α^2 for the two branches for $p = 1$ gravity in $d = 8$. The lower branch plot of the $\alpha^2 = 0.0008$ case is not reported because it is very close to the flat solution. *Idem* [Figure 6] *Idem* Fig. 1 for $p = 2$ gravity in $d = 6$.
- Figure 7 *Idem* Fig. 2 for $p = 2$ gravity in $d = 6$,
- Figure 8 The evolutions of $N(r)$, $\sigma(r)$, $w(r)$, at the approach of the critical value for $p = 2$ gravity in $d = 6$.
- Figure 9 *Idem* Fig. 1 for $p = 2$ gravity in $d = 8$.
- Figure 10 *Idem* Fig. 2 for $p = 2$ gravity in $d = 8$.
- Figure 11 *Idem* Fig. 8 for $p = 2$ gravity in $d = 8$.

Figure 1

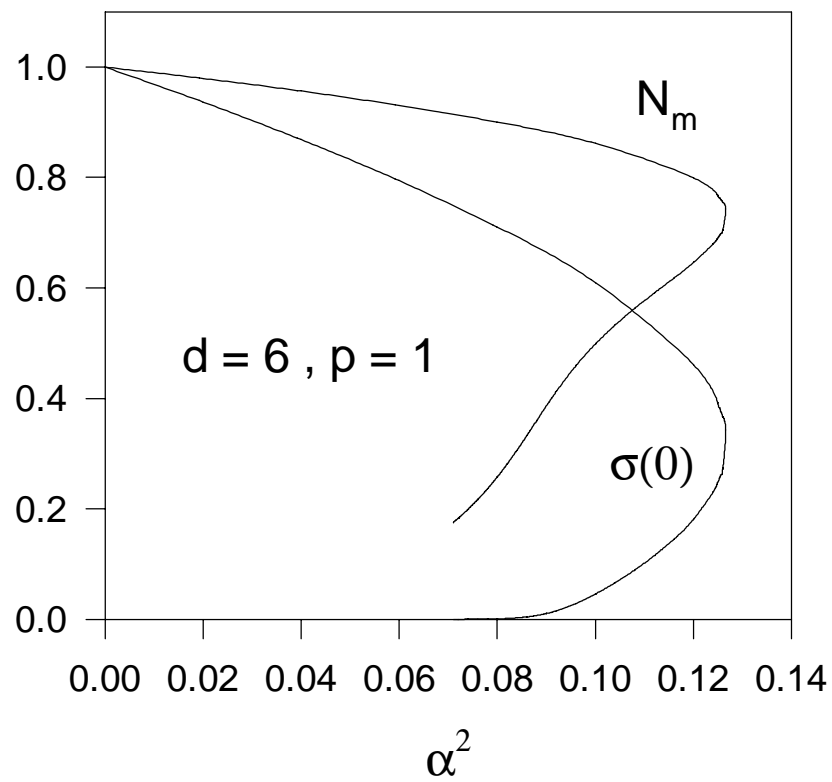


Figure 2

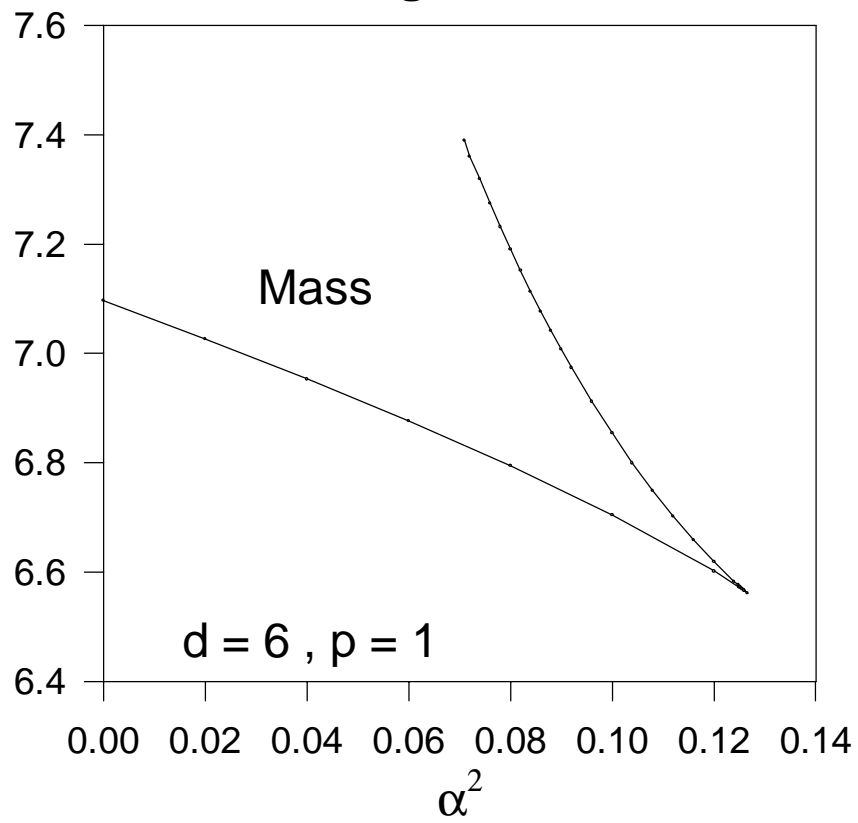


Figure 3

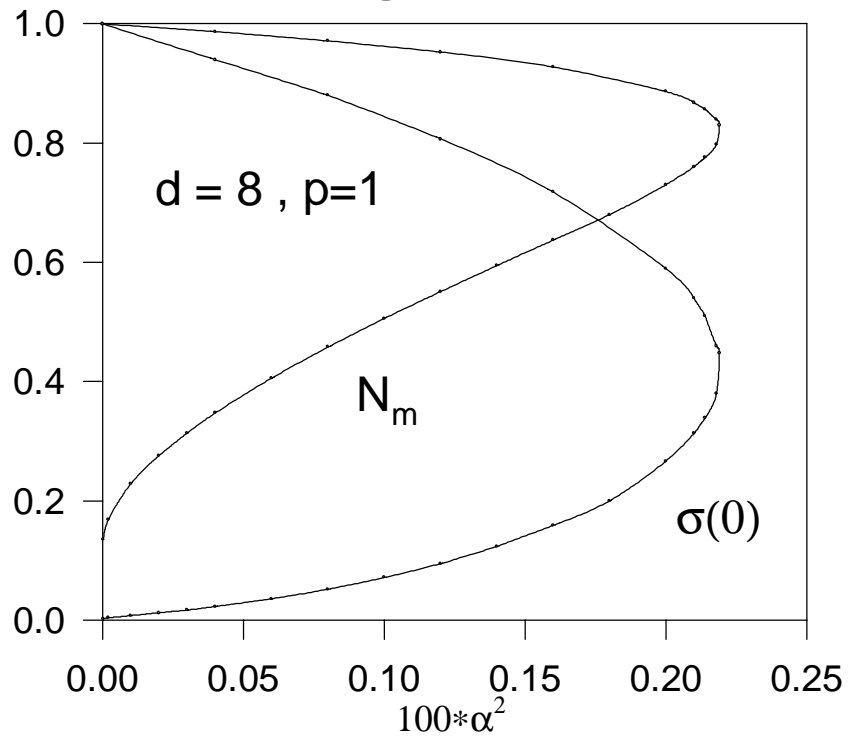


Figure 4

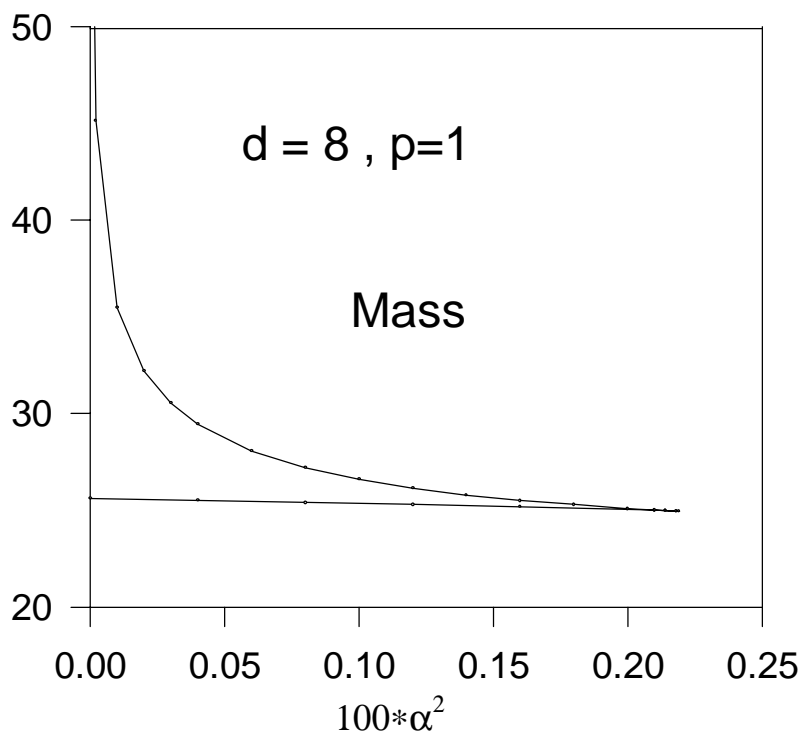


Figure 5

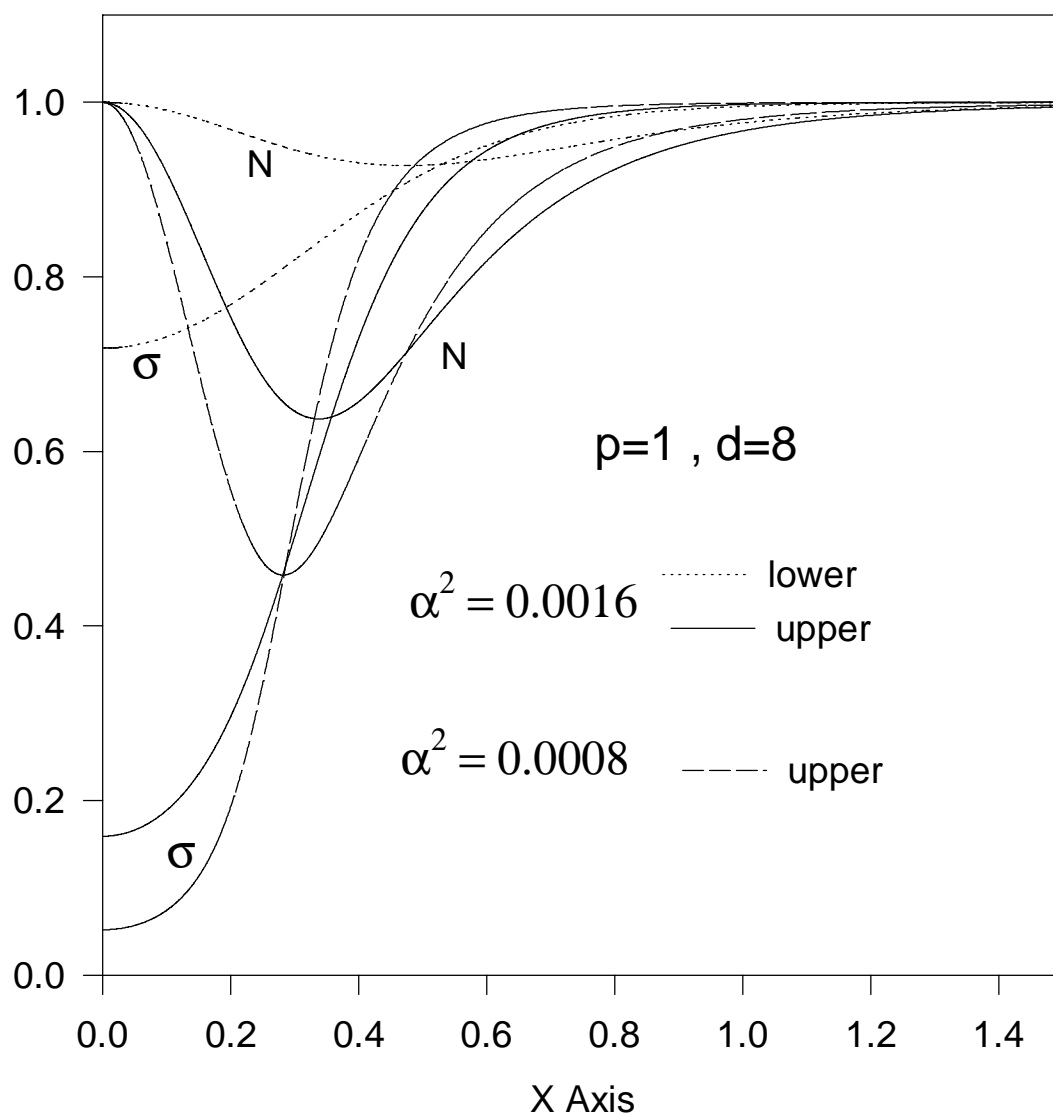


Figure 6

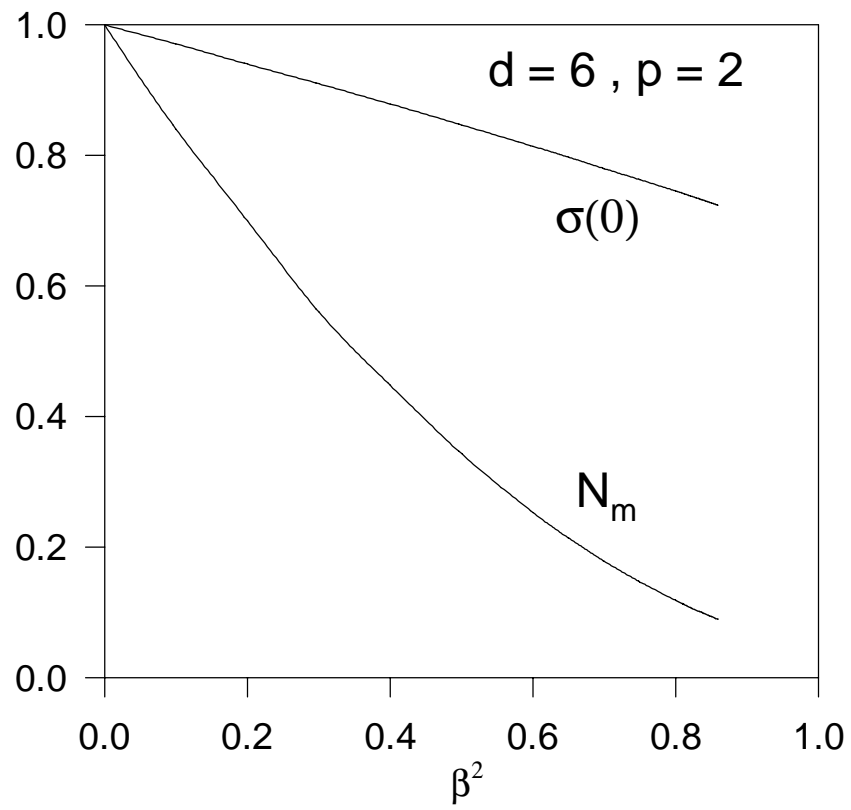


Figure 7

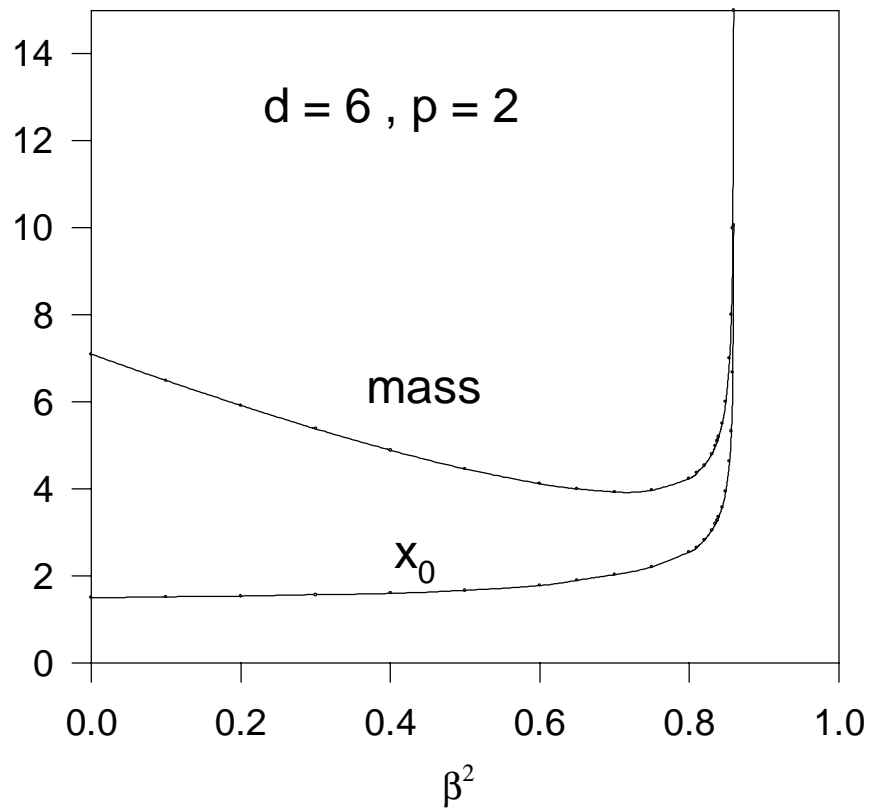


Figure 9

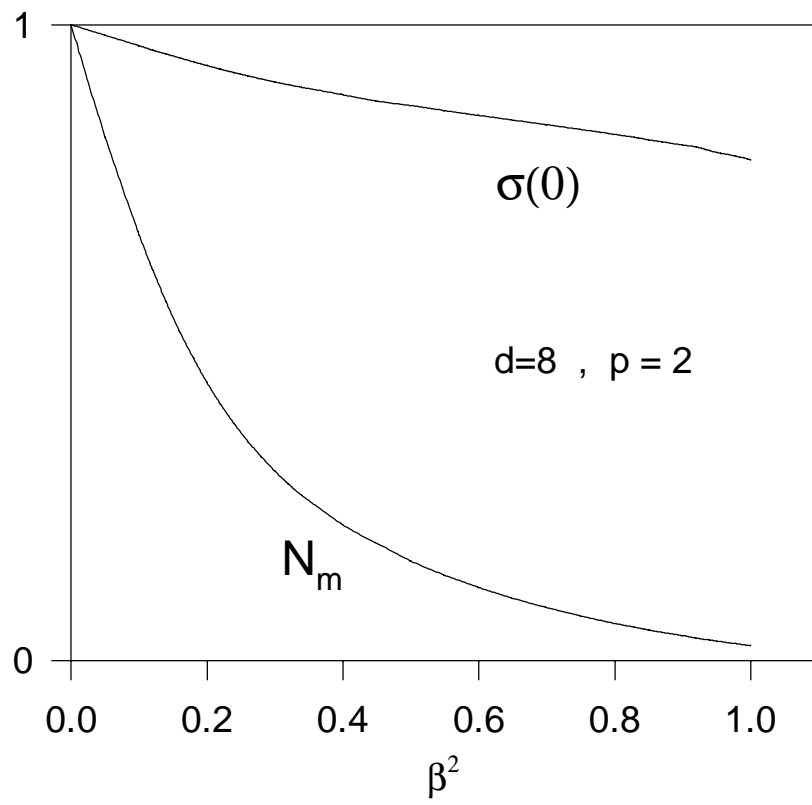


Figure 10

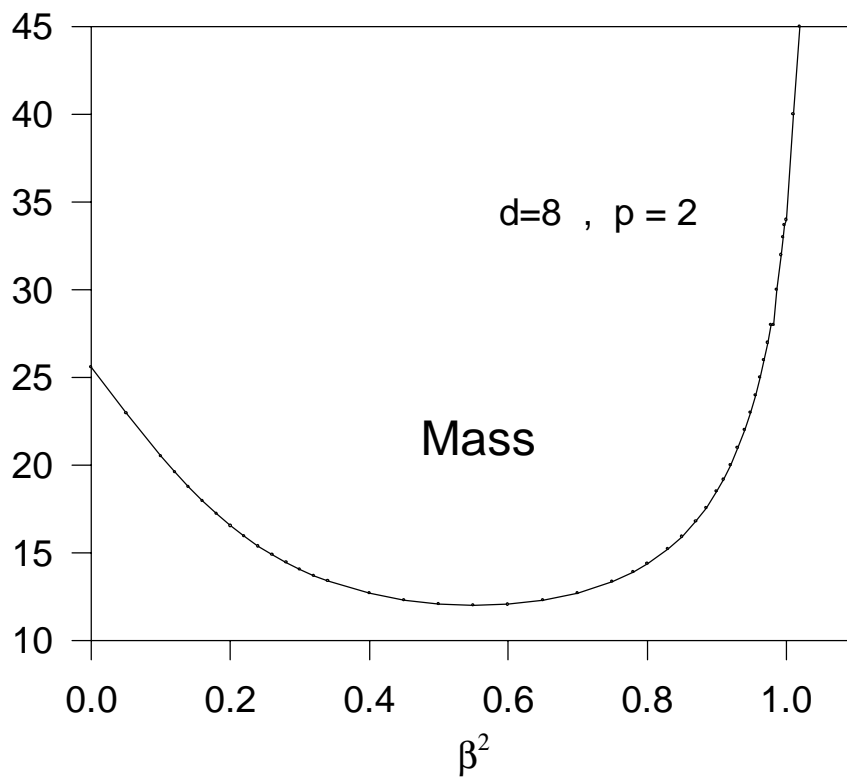


Figure 11

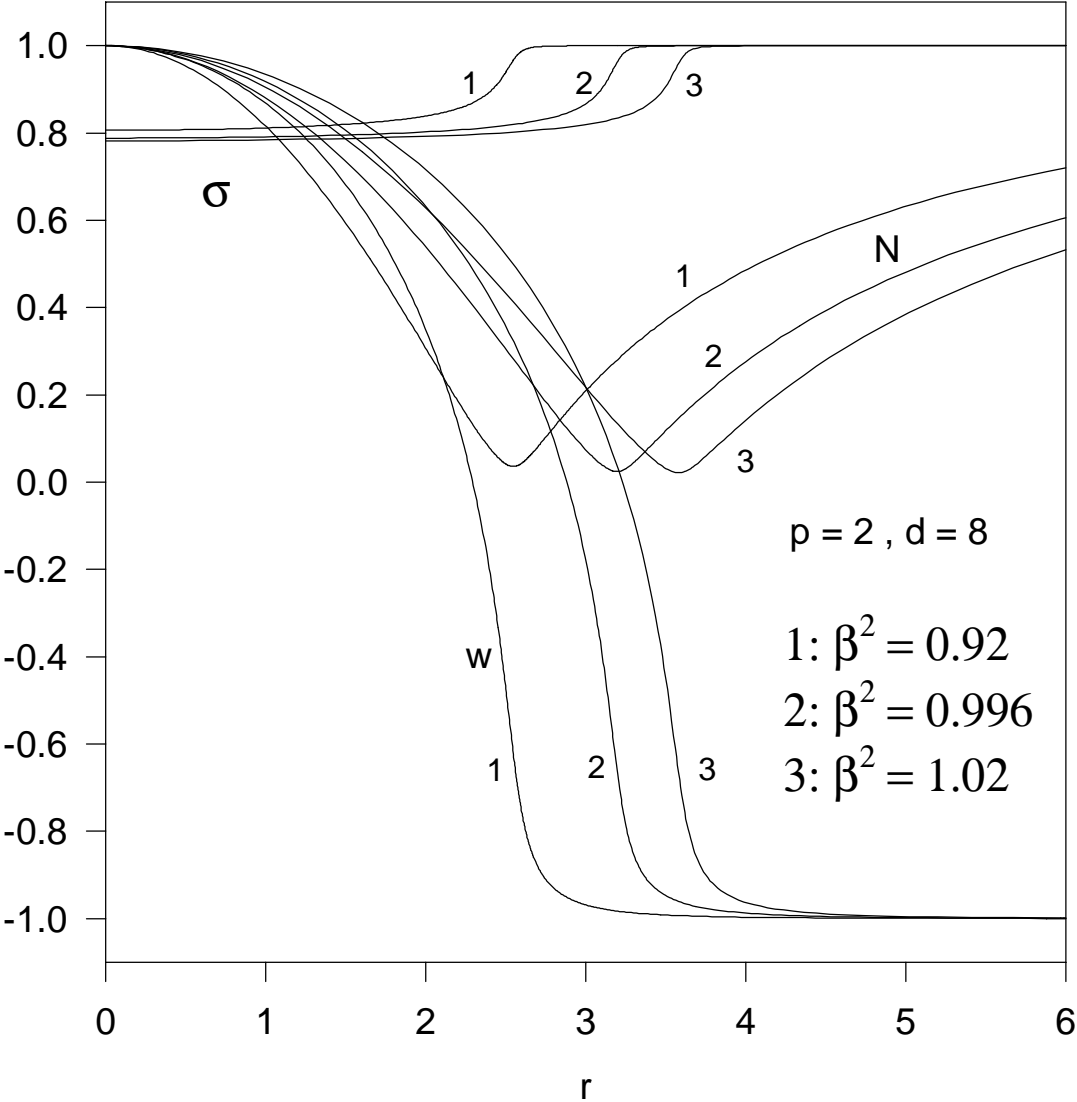


Figure 8

